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ON THE ACCOUSTIC FIELD OF A POINT SOURCE IN A RECTANGULAR SPACE BOUNDED BY THIN ELASTIC WALLS

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An analytic formula is obtained for Green's function of the Helmholtz equation in a rectangle. Along the rectangle sides boundary conditions containing higher order derivatives, and at its vertices boundary contact conditions are specified. Such boundary contact problem occurs in investigations of the acoustic field in a rectangular space bounded by thin elastic walls. The present paper is a continuation of papers [1, 2] where similar problems for the quarter-plane and half-band were considered.

1. **Statement of the problem and examples.** We seek a solution of the two-dimensional Helmholtz equation

$$(\Delta + k^2)P(x, y) = -\delta(x - x_0, y - y_0) \tag{1.1}$$

in the rectangle $0 < x < a$, $0 < y < b$ with boundary conditions

$$L_1\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) P(x, 0) = 0, \quad L_2\left(\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}\right) P(x, b) = 0 \qquad 0 < x < a \tag{1.2}$$

$$L_3\left(\frac{\partial}{\partial y}, \frac{\partial}{\partial x}\right) P(0, y) = 0, \quad L_4\left(\frac{\partial}{\partial y}, -\frac{\partial}{\partial x}\right) P(a, y) = 0 \qquad 0 < y < b \tag{1.3}$$

where P is the acoustic pressure in the medium, Δ is the Laplace operator, and k is the wave number which we assume to be complex $(0 < \arg k < \pi/4)$, thus allowing for absorption in the medium. The solution obtained below is, however, also valid in the case of perfect medium $(\arg k = 0)$ but under condition that k^2 is not an eigenvalue of the problem. The dependence on time is taken in the form of factor $e^{-i\omega t}$. The boundary operators L_α $(\alpha = 1, 2, 3, 4)$ are linear differential operators of order N_α with constant coefficients. We define their form by the formula

$$L_\alpha(\xi, \eta) = \eta m_{\alpha 1}(-\xi^2) + m_{\alpha 2}(-\xi^2) \tag{1.4}$$

where $m_{\alpha 1}$ and $m_{\alpha 2}$ are polynomials on which certain restrictions will be subsequently imposed.

The sought solution must be continuous in the considered region up to its boundary, except at the source point (x_0, y_0).

In the case of the simplest Dirichlet boundary conditions $(L_\alpha = 1)$ or those of Neumann $(L_\alpha = \eta)$ the stated problem has a unique solution which can be obtained elementarily by expansion in Fourier series or by the method of images. When the order of boundary operators exceeds unity, the solution looses its uniqueness, and contains N arbitrary constants, whose number can be calculated by formulas

$$N = N_{13} + N_{14} + N_{23} + N_{24} \tag{1.5}$$

$$N_{\alpha\beta} = E\left(\frac{N_\alpha + N_\beta - 1}{2}\right)$$

$$(\alpha = 1,\ 2;\ \beta = 3,\ 4)$$

where $E(x)$ is the integral part of the number x.

The arbitrariness in the solution is eliminated by the inclusion in the problem formulation of N boundary contact conditions that specify the modes at the region corner points

$$R_{\alpha\beta s}P(x_\beta,\ y_\alpha) + R_{\beta\alpha s}P(x_\beta,\ y_\alpha) = 0 \quad (\alpha = 1,\ 2;\ \beta = 3,\ 4;\ s = 1,\ 2,\ \ldots\ N_{\alpha\beta})$$

where

$$R_{\alpha\beta s}P(x_\beta,\ y_\alpha) = \lim_{x\to x_\beta}\left[(-1)^{\alpha+1}\frac{\partial}{\partial y}r_{\alpha\beta s1}\left(-i\frac{\partial}{\partial x}\right) + r_{\alpha\beta s2}\left(-i\frac{\partial}{\partial x}\right)\right]P(x,\ y_\alpha)$$

$$R_{\beta\alpha s}P(x_\beta,\ y_\alpha) = \lim_{y\to y_\alpha}\left[(-1)^{\beta+1}\frac{\partial}{\partial x}r_{\beta\alpha s1}\left(-i\frac{\partial}{\partial y}\right) + r_{\beta\alpha s2}\left(-i\frac{\partial}{\partial y}\right)\right]P(x_\beta,\ y)$$

$$x_3 = 0,\ x_4 = a,\ y_1 = 0,\ y_2 = b$$

and $r_{\alpha\beta sj}$ and $r_{\beta\alpha sj}$ $(j = 1,\ 2)$ are polynomials of their own arguments.

Example 1. The acoustic field of a point source in a rectangular region bounded by thin plates flexurally oscillating. In this case for the boundary operators we have

$$L_\alpha(\xi,\eta) = \eta(\xi^4 - \varkappa_\alpha^4) + \nu_\alpha, \quad \nu_\alpha = \rho\omega^2/D_\alpha \quad (\alpha = 1,\ 2,\ 3,\ 4)$$

where $\varkappa_\alpha$ is the wave number of flexural waves in a plate, ρ is the density of the medium, and D_α is the plate cylindrical stiffness.

For the complete definition of a particular model it is necessary to know the mechanical conditions at plate joints and corner points of regions (weld, hinge, joint, crack, etc.). At each vertex of the rectangle the mode is determined by four boundary contact equalities, hence in conformity with (1.5) we have here $N = 16$. We shall assume that the plates are rigidly soldered. In such case the set of boundary contact equalities for each of the corner points $(x_\beta,\ y_\alpha)$ $(\alpha = 1,\ 2;\ \beta = 3,\ 4)$ is of the form

$$\lim_{x\to x_\beta}\frac{\partial P(x,\ y_\alpha)}{\partial y} = 0, \quad \lim_{y\to y_\alpha}\frac{\partial P(x_\beta,\ y)}{\partial x} = 0$$

$$\lim_{x\to x_\beta}\frac{\partial^2 P(x,\ y_\alpha)}{\partial x\,\partial y} + \lim_{y\to y_\alpha}\frac{\partial^2 P(x_\beta,\ y)}{\partial y\,\partial x} = 0$$

$$(-1)^\alpha D_\alpha\lim_{x\to x_\beta}\frac{\partial^3 P(x,\ y_\alpha)}{\partial x^2\,\partial y} - (-1)^\beta D_\beta\lim_{y\to y_\alpha}\frac{\partial^3 P(x_\beta,\ y)}{\partial y^2\,\partial x} = 0$$

The first two equalities indicate the absence of plate dislocation at their joints, and the third implies the invariability of the angle between the plates, and the fourth the absence of torque at the plate joints.

Examples of other possible boundary operators are given in [3]. Taking into account longitudinal motions of plates would lead to operators L_α of the seventh order, which would necessitate the specification of six boundary contact conditions at each of the rectangle vertices.

E x a m p l e 2. The problem of sound transmittance through a partition separating two identical rectangular rooms. A thin elastic partition separates a rectangular room bounded by thin elastic walls in two compartments. The acoustic field induced by a point source acting in one of the compartments is to be determined.

We direct the Oy -axis along the partition. The problem reduces to the derivation of solution of the Helmholtz equation (1. 1) $(0 < |x| < a,\ 0 < y < b)$ with boundary conditions (1. 2) satisfied for $(0 < |x| < a)$; with boundary condition

$$L_3\left(\frac{\partial}{\partial y},\ \pm\frac{\partial}{\partial x}\right)P(\mp a, y) = 0 \quad (0 < y < b)$$

merging conditions

$$\frac{1}{2}m_{41}\left(-\frac{\partial^2}{\partial y^2}\right)\left[\frac{\partial P(+0, y)}{\partial x} + \frac{\partial P(-0, y)}{\partial x}\right] + m_{42}\left(-\frac{\partial^2}{\partial y^2}\right)[P(+0, y) - P(-0, y)] = 0$$

$$\frac{1}{2}m_{51}\left(-\frac{\partial^2}{\partial y^2}\right)\left[\frac{\partial P(+0, y)}{\partial x} - \frac{\partial P(-0, y)}{\partial x}\right] + m_{52}\left(-\frac{\partial^2}{\partial y^2}\right)[P(+0, y) + P(-0, y)] = 0$$

$$(0 < y < b)$$

and boundary contact conditions satisfied at point $(0, 0)$, $(0, b)$, $(\pm a, 0)$, and $(\pm a, b)$.

Dividing the field into the even and odd part with respect to the variable x

$$P(x, y) = P_+(x, y) + P_-(x, y), \quad P_\pm(x, y) = {}^1/_2\,[P(x, y) \pm P(-x, y)]$$

we decompose the problem into two, each of which can be considered in the region $0 < x < a,\ 0 < y < b$.

If only the flexural oscillations of the partition are taken into account, the merging conditions assume the form

$$\frac{1}{2}\left(\frac{\partial^4}{\partial x^4} - \varkappa_4^4\right)\left[\frac{\partial P(+0, y)}{\partial x} + \frac{\partial P(-0, y)}{\partial x}\right] + \nu_4[P(+0, y) - P(-0, y)] = 0$$

$$\frac{\partial P(+0, y)}{\partial x} = \frac{\partial P(-0, y)}{\partial x}$$

In this case the odd problem differs from that in Example 1 only by the doubling of some coefficients, and in the even problem when $x = 0$ we have the Neumann condition.

E x a m p l e 3. The problem of acoustics of a rectangular room divided by two perpendicular partitions into four identical compartments. Dividing the field into even and odd parts with respect to space variables reduces the problem to four independent problems of the considered here type.

Problems whose statement contains boundary conditions of the "second order"(boundary contact conditions) are usually called boundary contact problems [4]. The traditional procedure for finding an analytic solution of the boundary contact problem presumes the preliminary derivation of the general solution, i. e. a solution that satisfies all conditions of the problem, except the boundary contact ones. In two-dimensional problems such solution contain a certain number N of additive arbitrary constants (in the considered case N is determined by formulas (1. 5)). Such general solution can be presented as the sum of the particular solution P_0 of the non-homogeneous problem and of the general solution Q of the homogeneous problem.

The points of boundary at which the boundary contact conditions are imposed are called contact points. On the assumption that the point source does not act at contact points function P_0 can be selected so as to have continuous derivatives with respect of coordinates of all orders, which appear in the boundary contact conditions. Such selection of P_0 is single-valued. The obtained below explicit formula for P_0 implies in this case that this field has continuous derivatives of all orders at contact points.

The term Q represents the field radiating from the contact points. The field Q itslef is continuous in the considered region up to the boundary but carries in it discontinuities of derivatives of the complete field P at contact points.

To facilite the derivation of solution both components of the field are determined separately. It is advantageous to keep in mind the results of [1, 2] where solutions of similar problems were obtained.

2. T h e p a r t i c u l a r s o l u t i o n o f t h e n o n h o m o g e n e o u s p r o b l e m. Let us first consider the auxilliary problem of finding a solution of the Helmholtz equation (1. 1) in the band $(|x| < \infty,\ 0 < y < b)$ with boundary conditions (1. 2) satisfied for all $x \in (-\infty, \infty)$. The solution is assumed to be exponentially decreasing as $|x| \to \infty$. In other words we have to determine Green's function for an infinite plane wave guide the motion of whose walls is defined by Eqs. (1. 2). We denote the sought solution by P_1 and expect it to be of the form

$$P_1(x, y) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \exp\left[i\lambda(x - x_0) - \gamma|y - y_0|\right] \frac{d\lambda}{\gamma} + \frac{1}{4\pi} \int_{-\infty}^{\infty} B_1(\lambda) \exp(i\lambda x - \gamma y)\, d\lambda + \frac{1}{4\pi} \int_{-\infty}^{\infty} B_2(\lambda) \exp\left[i\lambda x - \gamma(b - y)\right] d\lambda, \qquad \gamma = \sqrt{\lambda^2 - k^2} \tag{2.1}$$

Selection of the radical is traditional [1]. The first right-hand side term represents the fundamental solution of the Helmoltz equation (1. 1) for an infinite medium, and the second and third terms specify the waves reflected by the wave guide walls. The substitution of expression (2. 1) into boundary conditions (1. 2) yields a linear system for the determination of the unkown functions $B_1(\lambda)$ and $B_2(\lambda)$. After calculations we obtain

$$P_1(x, y) = \frac{1}{4\pi}\int_{-\infty}^{\infty} D(\lambda, y_-, y_+)\exp[i\lambda(x-x_0)]\,d\lambda \tag{2.2}$$

$$D(\lambda, y_-, y_+) = \frac{D_1(\lambda, y_-)\,D_2(\lambda, y_+)}{D_{12}(\lambda)}$$

$$\gamma D_{12}(\lambda) = l_1 l_2 \exp(\gamma b) - l_1^\circ l_2^\circ \exp(-\gamma b)$$

$$\gamma D_1(\lambda, y) = l_1 \exp(\gamma y) - l_1^\circ \exp(-\gamma y)$$

$$\gamma D_2(\lambda, y) = l_2 \exp[\gamma(b-y)] - l_2^\circ \exp[-\gamma(b-y)]$$

$$l_\alpha = l_\alpha(\lambda) = -\gamma m_{\alpha 1}(\lambda^2) + m_{\alpha 2}(\lambda^2)$$

$$l_\alpha^\circ = l_\alpha^\circ(\lambda) = \gamma m_{\alpha 1}(\lambda^2) + m_{\alpha 2}(\lambda^2)$$

$$2y_\pm = y + y_0 \pm |y - y_0| \quad (\alpha = 1, 2)$$

Functions $D_{12}(\lambda)$, $D_1(\lambda, y)$, and $D_2(\lambda, y)$ are even integral functions of the variable λ. We assume that for $0 < \arg k < \pi/4$ function $D_{12}(\lambda)$ has no real roots. This limitation imposed on the properties of functions $l_1(\lambda)$ and $l_2(\lambda)$ is obvious from the physical point of view, since real roots of the dispersion equation correspond to wave numbers of regular undamped waves which are not possible in a wave guide filled by an absorbing medium. If $\arg k = 0$, i. e. in the case of a wave guide filled by a perfect medium, some of the roots of $D_{12}(\lambda)$ may appear on the real axis. The integration path in (2. 2) must be shifted in such case from the real axis so that the respective poles are bypassed in conformity with the principle of limit absorption.

We transform the integral in (2. 2) into a sum using the theorem on residues

$$P_1(x, y) = \frac{i}{2}\sum_s D_*(\lambda_s, y_-, y_+)\exp[i\lambda|x - x_0|] \tag{2.3}$$

$$D_*(\lambda, y_-, y_+) = \frac{D_1(\lambda, y_-)\,D_2(\lambda, y_+)}{dD_{12}(\lambda)/d\lambda}$$

where λ_s are roots of function $D_{12}(\lambda)$ which for $0 < \arg k < \pi/4)$ lie in the upper half-plane.

It is possible to eliminate symbols y_- and y_+ from formula (2. 3).
From the definition of λ_s as the root of $D_{12}(\lambda)$ follow the following equalities:

$$\frac{D_2(\lambda_s, y)}{l_2^\circ(\lambda_s)}\exp[\gamma(\lambda_s)b] = -\frac{D_1(\lambda_s, y)}{l_1(\lambda_s)}$$

$$\frac{D_2(\lambda_s, y)}{l_2(\lambda_s)}\exp[-\gamma(\lambda_s)b] = -\frac{D_1(\lambda_s, y)}{l_1^\circ(\lambda_s)}$$

Using, as an example, the first of these, we obtain the relationships

$$D_*(\lambda_s, y_-, y_+) = D_*(\lambda_s\, y, y_0) = D_*(\lambda_s, y_0, y)$$

We shall use for P_1 the representation (2. 3). To obtain the sought field P_0 it is necessary to add to P_1 the waves reflected from the side walls $x = 0$ and $x = a$ of the considered space

$$P_0(x, y) = P_1(x, y) + \frac{i}{2}\sum_s A_1(\lambda_s) D_*(\lambda_s, y_-, y_+) \exp(i\lambda_s x) + \frac{i}{2}\sum_s A_2(\lambda_s) D_*(\lambda_s, y_-, y_+) \exp[i\lambda_s(a - x)] \tag{2.4}$$

The substitution of (2. 4) into boundary conditions (1. 3) yields a system of linear equations for the determination of the reflection coefficients $A_1(\lambda_s)$ and $A_2(\lambda_s)$. After some transformations we obtain

$$P_0(x, y) = \frac{1}{2}\sum_s \lambda_s D_*(\lambda_s, y_-, y_+) \Delta(\lambda_s, x_-, x_+) \tag{2.5}$$

$$\Delta(\lambda, x_-, x_+) = \frac{\Delta_3(\lambda, x_-)\, \Delta_4(\lambda, x_+)}{\Delta_{34}(\lambda)}$$

$$i\lambda\Delta_{34}(\lambda) = n_3 n_4 \exp(-i\lambda a) - n_3^\circ n_4^\circ \exp(i\lambda a)$$

$$i\lambda\Delta_3(\lambda, x) = n_3 \exp(-i\lambda x) - n_3^\circ \exp(i\lambda x)$$

$$i\lambda\Delta_4(\lambda, x) = n_4 \exp[-i\lambda(a - x)] - n_4^\circ \exp[i\lambda(a - x)]$$

$$n_\alpha = n_\alpha(\lambda) = i\lambda m_{\alpha 1}(k^2 - \lambda^2) + m_{\alpha 2}(k^2 - \lambda^2)$$

$$n_\alpha^\circ = n_\alpha^\circ(\lambda) = n_\alpha(-\lambda) \quad (\alpha = 3, 4)$$

$$2x_\pm = x + x_0 \pm |x - x_0|$$

Functions $n_3(\pm\lambda)$ and $n_4(\pm\lambda)$ are polynomials and $\Delta_{34}(\lambda)$, $\Delta_3(\lambda, x)$, and $\Delta_4(\lambda, x)$ are even integral functions of the variable λ. Functions $D_{12}(\lambda)$ and $\Delta_{34}(\lambda)$ have no common roots. This restriction on the properties of boundary operators L_α $(\alpha = 1, 2, 3, 4)$ is reasonable, since otherwise a resonance conversion of the field into infinity would occur in spite of absorption in the medium.

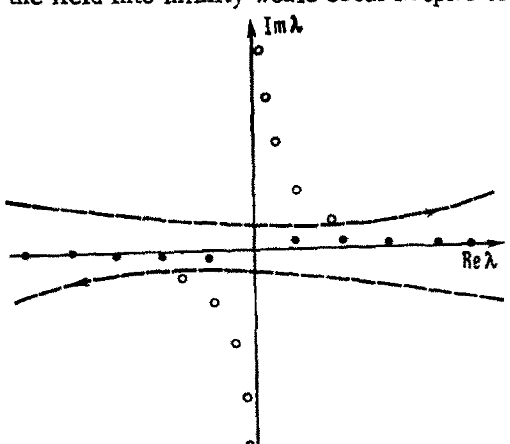

Fig. 1

The disposition of roots of functions $D_{12}(\lambda)$ and $\Delta_{34}(\lambda)$ is diagrammatically shown in Fig. 1 by small circules and dots, respectively. In the coordinate origin neighborhood the distribution of roots is irregular, it is determined by the particular specification of boundary operators. The roots $D_{12}(\lambda)$ asymptotically approach roots sh γb, as $|\lambda| \to \infty$, and roots $\Delta_{34}(\lambda)$ asymptotically tend to roots $\sin \lambda a$.

The equality (2. 5) defines the

expansion of function $P_0(x, y)$ in a Fourier series in eigenfunctions of the Sturm — Liouville problem for the variable y. Using the theorem on residues we pass to the integral representation of $P_0(x, y)$

$$P_0(x, y) = \frac{1}{8\pi i}\int\limits_{\Lambda} D(\lambda, y_-, y_+)\Delta(\lambda, x_-, x_+)\lambda\, d\lambda \tag{2.6}$$

where Λ is the contour which divides the roots of functions $D_{12}(\lambda)$ and $\Delta_{34}(\lambda)$ and the roots of $D_{12}(\lambda)$ lie on the left of the contour run, and consists of two branches that are symmetric about the coordinate origin. In Fig. 1 the contour Λ is shown by dash lines. Owing to the oddness of the integrand formula (2.6) is unaffected by the change of sign of the variable of integration. It is convenient because it maintains the equilvalence of the space variables x and y. Thus using the theorem on residues for the region comprised between the bránches of contour Λ we revert to representation (2.5), and applying the same reasoning to the region outside the branches of contour Λ we obtain an analogous expansion in which appear the eigen functions of the Sturm — Liouville problem for the variable x.

3. General solution of the homogeneous problem. In this case the general solution Q of the homogeneous problem is of the form

$$Q = Q_{13} + Q_{14} + Q_{23} + Q_{24}$$

where $Q_{\alpha\beta}$ $(\alpha = 1, 2;\ \beta = 3, 4)$ is the field radiating from the corner point (x_β, y_α) of the considered region. For simplicity we derive the analytic expression for Q by analogy to the corresponding components of the field in problems considered in [1, 2].

Green's function of the Helmholtz equation for region $x > 0$ and $y > 0$ was obtained in [1] under boundary and boundary contact conditions similar to those considered here. We use coordinates (x_0, y_0) of the point of application of the source as the separate arguments of function P_0. The particular solution of the nonhomogeneous problem in [1] in the notation used here is of the form of the sum of source fields and three of its representations

$$P_0(x, y, x_0, y_0) = \frac{1}{4\pi}\int\limits_{\Lambda_1}\Big\{\exp[i\lambda(x - x_0) - \gamma|y - y_0|] -$$

$$\frac{l_1^\circ}{l_1}\exp[i\lambda(x - x_0) - \gamma(y + y_0)] - \frac{n_2^\circ}{n_2}\exp[i\lambda(x + x_0) -$$

$$\gamma|y - y_0|] + \frac{l_1^\circ n_2^\circ}{l_1 n_2}\exp[i\lambda(x + x_0) - \gamma(y + y_0)]\Big\}\frac{d\lambda}{\gamma}$$

where Λ_1 is a certain contour that separates roots l_1 which lie in the upper half-plane from the remaining roots of functions l_1 and n_2. Let us define the form of $P_0(x, y, x_0, y_0)$ when the field source is transferred to the contact point. We have

$$P_0(x, y, 0, 0) = -\frac{i}{\pi}\int\limits_{\Lambda_1}\frac{\lambda m_{11}(\lambda^2)\, m_{21}(k^2 - \lambda^2)}{l_1 n_2}\, e^{i\lambda x - \gamma y}\, d\lambda \tag{3.1}$$

Let us compare this formula with the corresponding solution of the homogeneous problem in [1]

$$Q(x, y) = \frac{1}{4\pi}\int\limits_{\Lambda_1} \frac{\lambda q_{N-1}(\lambda^2)}{l_1 n_2} e^{i\lambda x - \gamma y}\, d\lambda$$

where $q_{N-1}(\lambda^2)$ is an even polynomial of $2N-2$ power with arbitrary coefficients. In (3. 1) instead of $q_{N-1}(\lambda^2)$ we have an even polynomial of power higher by two, and known coefficients. A similar correspondence exists between P_0 and Q_α ($\alpha = 1, 2$) in the problem solved in [2] (the region considered in [2] was a semi-infinite plane wave guide $0 < x < \infty$, $0 < y < b$, with two contact points).

Let us revert to our problem. After passing to the limit $(x_0, y_0) \to (0,0)$ we obtain

$$P_0(x, y, 0, 0) = -\frac{1}{2\pi}\int\limits_{\Lambda} \frac{D_2(\lambda, y)\,\Delta_4(\lambda, x)}{D_{12}(\lambda)\,\Delta_{34}(\lambda)} \lambda m_{11}(\lambda^2)\, m_{21}(k^2 - \lambda^2)\, d\lambda$$

Hence the field Q_{13} should be expected to be of the form

$$Q_{13}(x, y) = \frac{1}{8\pi i}\int\limits_{\Lambda} \frac{D_2(\lambda, y)\,\Delta_4(\lambda, x)}{D_{12}(\lambda)\,\Delta_{34}(\lambda)} \lambda q_{N_{13}-1}(\lambda^2)\, d\lambda \tag{3. 2}$$

Since this formula was obtained on the basis of certain analogies, and not by a systematic procedure, it needs verification. The validity of the following relationships:

$$(\Delta + k^2) D_2(\lambda, y)\Delta_4(\lambda, x) = 0$$

$$L_1\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) D_2(\lambda, 0)\,\Delta_4(\lambda, x) = D_{12}(\lambda)\,\Delta_4(\lambda, x)$$

$$L_2\left(\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}\right) D_2(\lambda, b)\,\Delta_4(\lambda, x) = 0$$

$$L_3\left(\frac{\partial}{\partial y}, \frac{\partial}{\partial x}\right) D_2(\lambda, y)\,\Delta_4(\lambda, 0) = D_2(\lambda, y)\,\Delta_{34}(\lambda)$$

$$L_4\left(\frac{\partial}{\partial y}, -\frac{\partial}{\partial x}\right) D_2(\lambda, y)\,\Delta_4(\lambda, a) = 0$$

can be proved by direct differentiation.

It is, thus, evident that the homogeneous Helmoltz equation and of the second boundary conditions of (1. 2) and (1. 3) are satisfied. Let us consider the remaining two boundary conditions. We have

$$L_1\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) Q_{13}(x, 0) = \frac{1}{8\pi i}\int\limits_{\Lambda} \frac{\Delta_4(\lambda, x)}{\Delta_{34}(\lambda)} \lambda q_{N_{13}-1}(\lambda^2)\, d\lambda$$

$$L_3\left(\frac{\partial}{\partial y}, \frac{\partial}{\partial x}\right) Q_{13}(0, y) = \frac{1}{8\pi i}\int\limits_{\Lambda} \frac{D_2(\lambda, x)}{D_{12}(\lambda)} \lambda q_{N_{13}-1}(\lambda^2)\, d\lambda$$

For $0 < x < a$ the integrand in the first of these equalities exponentially decreases as $|\operatorname{Im}\lambda| \to \infty$ and has no singularities outside the branches of contour Λ. Using the theorem on residues for the region outside the branches of contour Λ we obtain that $L_1 Q_{13}(x, 0) = 0$. The equality $L_3 Q_{13}(0, y) = 0$ follows similarly from the theorem on residues applied to the region comprised between the branches of contour Λ.

The continuity of Q_{13} in the coordinate origin neighborhood is implied by the uniform convergence of the integral in (3.2).

Expressions for Q_{14}, Q_{23}, and Q_{24} can be derived from (3.2) by cyclic replacement of subscripts 1, 2 and 3, 4.

Thus the general solution of the homogeneous problem has been derived. Formula (3.2) contains the necessary number N_{13} of arbitrary constants, because of which, using boundary contact conditions, a linear system is obtained for the determination of these constants, which contains the same number of equations and unknowns. The formal application of like boundary operators $R_{\alpha\beta s}$ and $R_{\beta\alpha s}$ to $Q_{\alpha\beta}$ ($\alpha = 1, 2$ and $\beta = 3, 4$) generally results in divergent integrals. To make the regularization of these integrals possible the boundary contact operators must satisfy certain special requirements. The respective conditions were stated in [1, 2]. When these conditions are satisfied, regularization can be carried out by the method described in [2].

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